H

National
Qualifications
2022 MODIFIED

X847/76/11

Mathematics
Paper 1 (Non-calculator)

FRIDAY, 6 MAY

9:00 AM – 10:15 AM

Total marks — 55

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Total marks — 55

Attempt ALL questions

1. Determine the equation of the line perpendicular to $5x+2y=7$, passing through $(-1,6)$. **3**

2. Evaluate $2\log_3 6-\log_3 4$. **3**

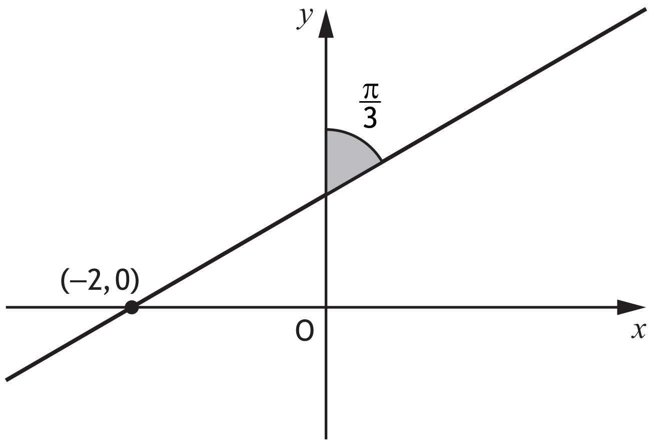
3. A function, h, is defined by $h(x)=4+\frac{1}{3}x$, where $x\in\mathbb{R}$.
Find the inverse function, $h^{-1}(x)$. **3**

4. Differentiate $y=\sqrt{x^3}-2x^{-1}$, where $x>0$. **3**

[Turn over

MARKS

5. A line makes an angle of $\frac{\pi}{3}$ radians with the y-axis, and passes through the point $(-2, 0)$ as shown below.



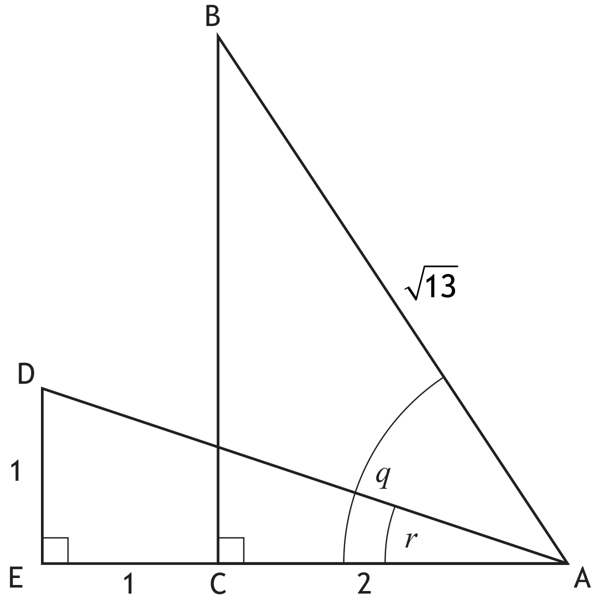
Determine the equation of the line. **3**

6. Evaluate $\int_{-5}^{2} (10 - 3x)^{-\frac{1}{2}}\, dx$. **4**

MARKS

7. Triangles ABC and ADE are both right angled.

Angle BAC $= q$ and angle DAE $= r$ as shown in the diagram.



(a) Determine the value of:

(i) $\sin r$ **1**

(ii) $\sin q$. **1**

(b) Hence determine the value of $\sin(q - r)$. **3**

8. Solve $\log_6 x + \log_6 (x+5) = 2$, where $x > 0$. **4**

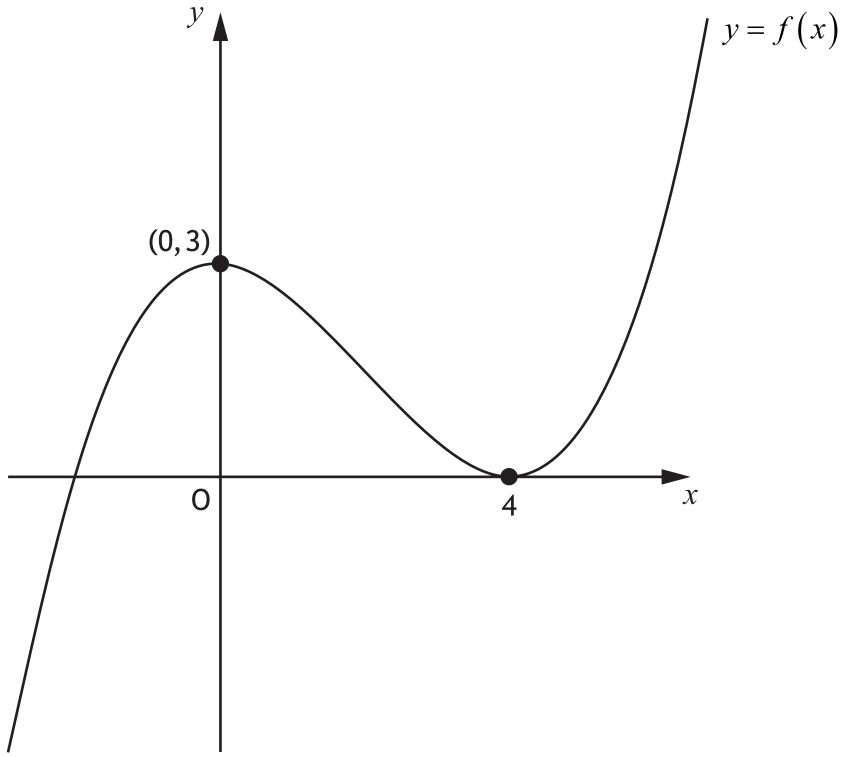
9. Solve the equation $\cos 2x° = 5\cos x° - 3$ for $0 \leq x < 360$. **5**

[Turn over

MARKS

10. The diagram shows the graph of a cubic function with equation $y = f(x)$.

The curve has stationary points at (0, 3) and (4, 0).



(a) Sketch the graph of $y = 2f(x) + 1$. **3**

Use the diagram provided in the answer booklet.

(b) State the coordinates of the stationary points on the graph of $y = f\left(\frac{1}{2}x\right)$. **1**

MARKS

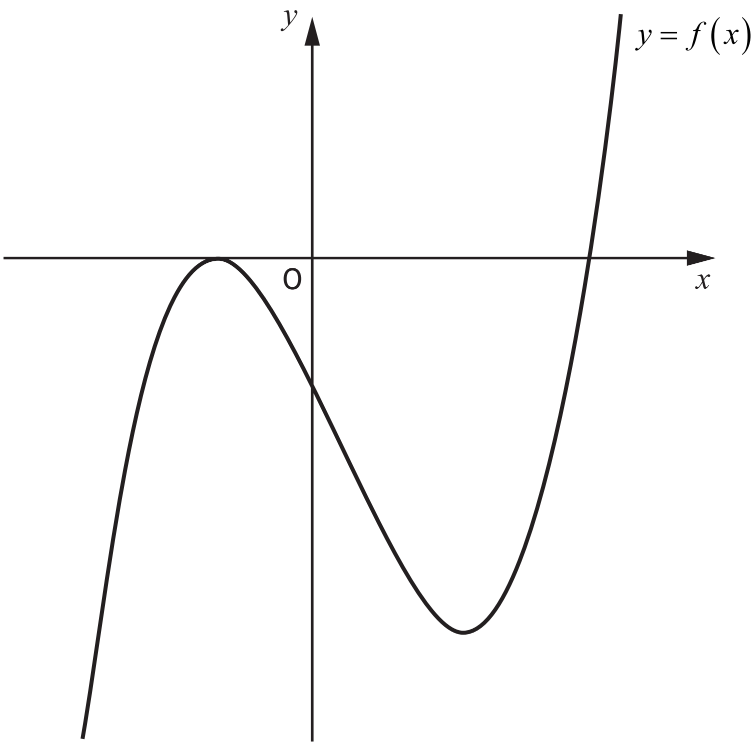
11. Express $2x^2 + 12x + 23$ in the form $p(x+q)^2 + r$. **3**

12. Given that $f(x) = 4\sin\left(3x - \frac{\pi}{3}\right)$, evaluate $f'\left(\frac{\pi}{6}\right)$. **3**

13. (a) (i) Show that $(x+2)$ is a factor of $f(x) = x^3 - 2x^2 - 20x - 24$. **2**

(ii) Hence, or otherwise, solve $f(x) = 0$. **3**

The diagram shows the graph of $y = f(x)$.



(b) The graph of $y = f(x - k)$, $k > 0$ has a stationary point at (1, 0).

State the value of k. **1**

[Turn over

MARKS

14. C_1 is the circle with equation $(x-7)^2+(y+5)^2=100$.

(a) (i) State the centre and radius of C_1. **2**

(ii) Hence, or otherwise, show that the point P$(-2, 7)$ lies outside C_1. **2**

C_2 is a circle with centre P and radius r.

(b) Determine the value(s) of r for which circles C_1 and C_2 have exactly one point of intersection. **2**

[END OF QUESTION PAPER]

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National
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X847/76/12

Mathematics
Paper 2

FRIDAY, 6 MAY

10:45 AM – 12:15 PM

Total marks — 65

Attempt ALL questions.

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FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
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Table of standard integrals

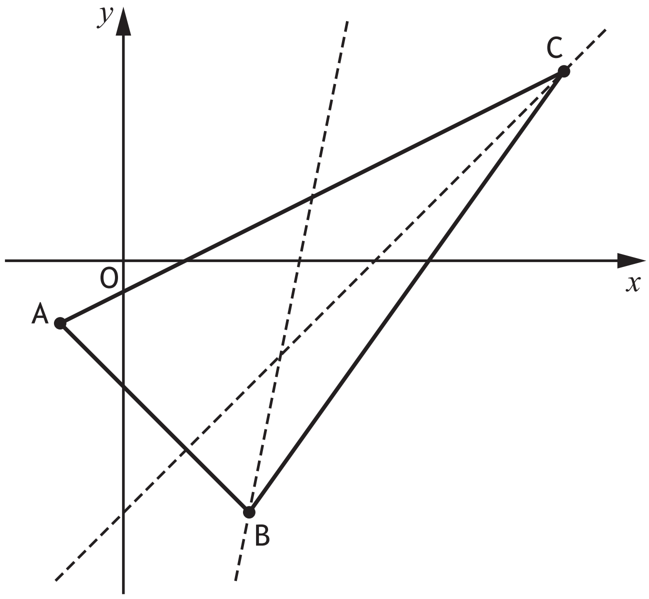
$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Total marks — 65

Attempt ALL questions

1. Triangle ABC has vertices A(–1, –1), B(2, –4) and C(7, 3).



(a) Find the equation of the altitude through C. **3**

(b) Find the equation of the median through B. **3**

(c) Determine the coordinates of the point of intersection of the altitude through C and the median through B. **2**

2. The equation $2x^2 - 8x + (4 - p) = 0$ has two real and distinct roots.

Determine the range of values for p. **3**

3. (a) Express $4\sin x + 5\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 < a < 2\pi$. **4**

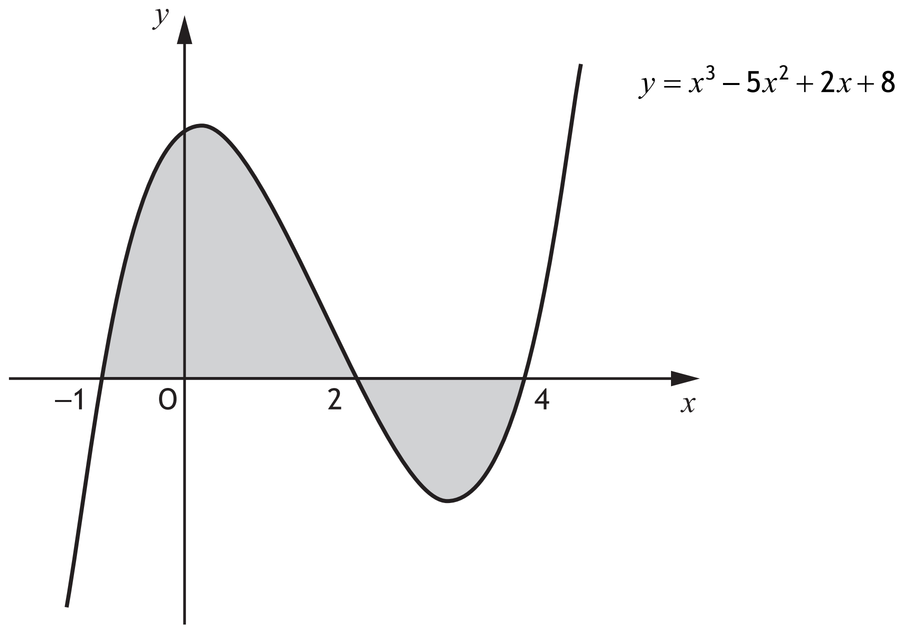
(b) Hence solve $4\sin x + 5\cos x = 5.5$ for $0 \le x < 2\pi$. **3**

[Turn over

MARKS

4. The graph shown has equation $y = x^3 - 5x^2 + 2x + 8$.

The total shaded area is bounded by the curve and the x-axis.



(a) Calculate the shaded area above the x-axis. **4**

(b) Hence calculate the total shaded area. **3**

5. Functions f and g are given by $f(x) = x^2 \quad 2$ and $g(x) = 3x \quad 5$, $x \in \mathbb{R}$.

(a) Find expressions for:

(i) $f(g(x))$ and **2**

(ii) $g(f(x))$. **1**

(b) Determine the range of values of x for which $f(g(x)) < g(f(x))$. **4**

MARKS

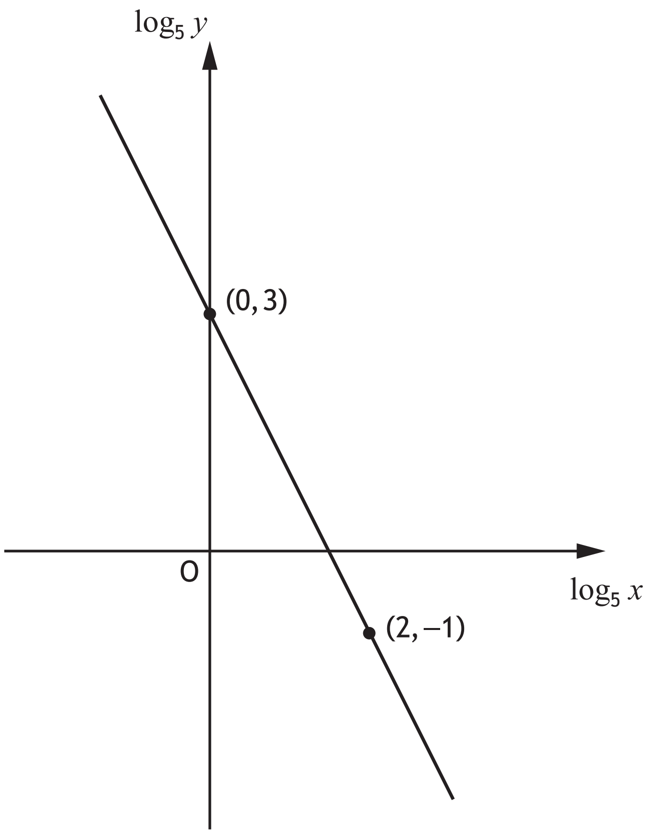
6. A curve with equation $y = f(x)$ is such that $\frac{dy}{dx} = 1 \quad \frac{3}{x^2}$, where $x > 0$.

The curve passes through the point $(3, 6)$.

Express y in terms of x. **5**

7. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.



Find the values of k and n. **5**

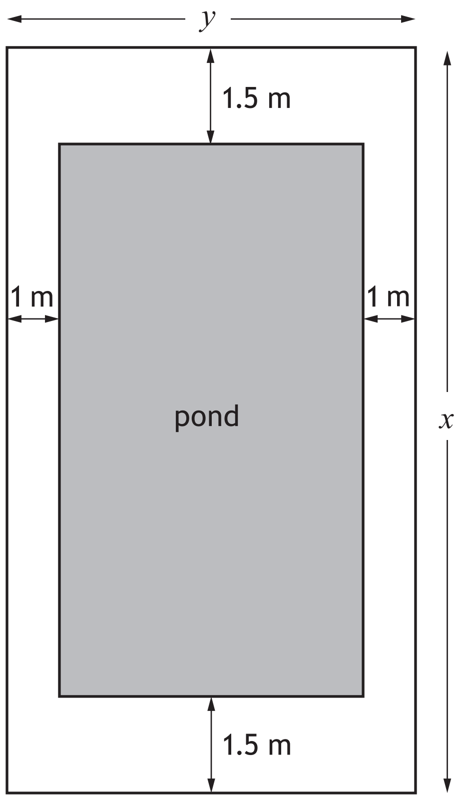
[Turn over

MARKS

8. A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



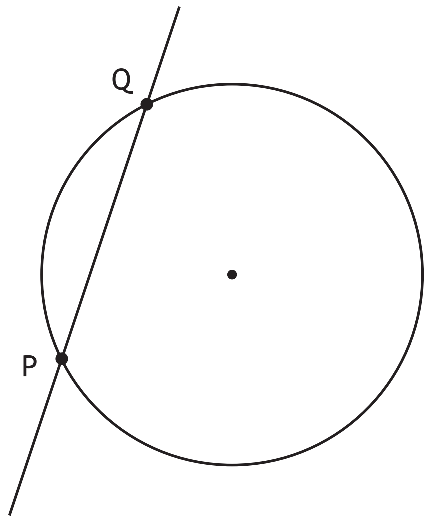
The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

$$A(x) = 156 - 2x - \frac{450}{x}.$$ **3**

(b) Determine the maximum area of the pond. **6**

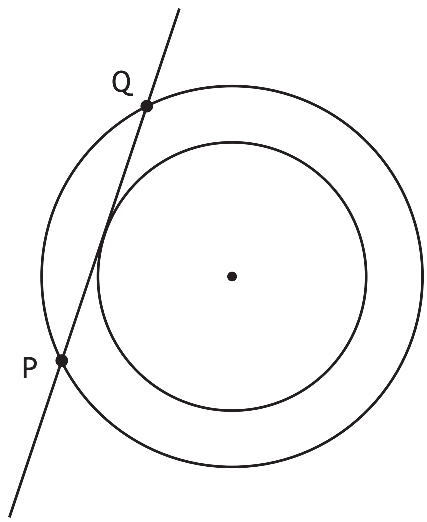
9. The line $y = 3x\quad 7$ intersects the circle $x^2 + y^2 - 4x - 6y - 7 = 0$ at the points P and Q.



(a) Find the coordinates of P and Q. **5**

PQ is a tangent to a second, smaller circle.

This circle is concentric with the first.



(b) Determine the equation of the smaller circle. **4**

[Turn over

MARKS

10. The heptathlon is an athletics contest made up of seven events.

Athletes score points for each event.

In the 200 metres event, the points are calculated using the formula

$$P = 4.99087(42.5 - T)^{1.81}$$

where P is the number of points awarded, and T is the athlete's time, in seconds.

(a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event. **1**

In the long jump event, the points are calculated using the formula

$$P = 0.188807(D - 210)^{k}$$

where P is the number of points awarded, D is the distance jumped, in centimetres, and k is a constant.

(b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k. **4**

[END OF QUESTION PAPER]